

Dirac-field model of inflation in Einstein-Cartan theory

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We present a cosmological model in which a single Dirac field with a flat potential can give rise to inflation within the framework of the Einstein-Cartan theory. It is shown that our Dirac-field model leads to a nearly scale-invariant spectrum of density fluctuations owing to the spin-interaction which naturally arises from the field equations of the Einstein-Cartan theory.

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I. INTRODUCTION

Nowadays, it is widely accepted that inflation of the early universe [1] is due to one or more slow-rolling scalar fields, called inflaton, which typically lead to a nearly scale-invariant power spectrum of primordial density fluctuations with the spectral index $n \approx 1$ in good agreement with recent, increasingly accurate observations of cosmic microwave background anisotropy [2, 3, 4, 5]. However, it remains a fundamental task to identify the inflaton with a certain particle in promising particle-physics models theoretically and experimentally. Then the question arises: cannot other fields drive inflation? Particularly we focus on Dirac spinor fields.

As the scalar field is often necessary in various theories of nature, so the Dirac field is essential to a satisfactory description of relativistic fermions. Thus, it is of considerable importance and interest to investigate roles of the Dirac field in cosmology.

On this subject, much remarkable work has been done within general relativity by various authors. For example, quantization of a Dirac field and studies of particle creation in the expanding Friedmann-Robertson-Walker (FRW) universe were presented in Refs. [6, 7, 8]; quantization of the system where a Dirac field couples to the FRW metric was discussed in Refs. [9, 10]; the explicit solutions of the Dirac equation in the FRW universe were derived in Refs. [11, 12].

Moreover, as for the issue of the accelerating universe, it was shown that a self-interacting or nonlinear Dirac field can yield negative pressure and thereby accelerate the early and the late-time expansion of the universe [13, 14]. However, when we discuss whether a Dirac field can be an alternative to the inflaton, we should pay attention to Armendáriz-Picón and Greene's work [15] which concluded that, although the de Sitter expansion can be driven by a flat potential of a Dirac field, such models lead to a scale-dependent spectrum with $n = 4$, and thus are incompatible with the observations. As other ways not to rely on the inflaton, one can introduce vec-

tor fields [16, 17], or non-Dirac spinors that have some unusual properties [18].

On the other hand, since Dirac fields have spin-1/2 in contrast to scalar fields, it is also of great interest to study cosmological effects of the spin. Then, one needs gravitational theories that naturally bring spin of matter fields to the geometry of spacetime, since in general relativity microscopic quantities such as the spin are usually neglected. For this purpose, as one of such extended theories, we will adopt the Einstein-Cartan theory [19] in which nonvanishing torsion is algebraically equivalent to spin of matter fields through the field equation and, as a result, a spin-interaction is generated. Generally, one can suppose affine connections in spacetime to be asymmetric, thereby naturally obtaining a modified theory of general relativity. Since, from the viewpoint of high energy physics, general relativity should appropriately be extended so as to be able to describe the very early universe, the Einstein-Cartan theory can be a reasonable framework for considering inflation. We can then expect that the spin has a crucial role in inflationary cosmology.

Many authors have also presented a variety of cosmological models based on non-Riemannian gravity [20, 21], as exhaustively listed in Ref. [22]. The subject of early investigations based on the Einstein-Cartan theory was mainly to construct exact solutions and to avoid the initial singularity in the presence of torsion [22, 23, 24]. Subsequently, after the advent of inflationary cosmology, inflationary solutions were obtained also in the Einstein-Cartan models, some of which [25, 26, 27, 28] relied on spin effects of the so-called spinning fluid [29, 30, 31].

In this paper, we show that a single Dirac field can give rise to inflation within the Einstein-Cartan theory, and prove compatibility of the Dirac-field model with the observations by calculating the power spectrum of density fluctuations of the Dirac field.

We will take the Dirac Lagrangian as the source of metric and torsion because we are only concerned with seeking the origin of inflation in the context of particle-physics. It should be noted that we will deal with the homogeneous Dirac field classically, namely, as a set of complex-valued functions that transform according to the spinor representation of the Lorentz group and fulfill the Dirac equation. We here consider the expectation value

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of a spinor operator-valued field to be the corresponding classical spinor; a possible justification for the existence of classical spinors is made in Ref. [15]. In addition to the fact that the classical spinors are mathematically consistent, as shown in Ref. [15], such classical treatment for a physical state can be a good approximation in some cases.

The organization of this paper is as follows: in section II, we briefly review the description of a Dirac field coupled to gravity in the Einstein-Cartan theory and derive cosmological equations in the flat FRW universe. Within this framework, in section III, after showing that a flat potential of the Dirac field leads to the de Sitter expansion of the universe, we compute the power spectrum of density fluctuations of the Dirac field. We will observe that the obtained spectrum is nearly scale-invariant owing to the existence of the spin-interaction. We present our conclusion in section IV.

II. FORMALISM

In this section, we summarize how the Einstein-Cartan theory describes interactions between a Dirac field and gravity, and present the cosmological equations that form the basis of our Dirac-field model of inflation.

In order to deal with a Dirac field ψ coupled to gravity, one requires that the action of the Dirac field be invariant under local Lorentz transformations, with the help of the vierbein formalism [32, 33] in which vierbeins e_μ^a and spin connections ω_μ^{ab} are introduced as the fundamental variables. The vierbein satisfies $\eta_{ab}e_\mu^ae_\nu^b = g_{\mu\nu}$ with the Minkowski metric η_{ab} , where the signature we use is $\eta_{ab} = \text{diag}(+1, -1, -1, -1)$. (The Greek and Latin indices denote spacetime and Lorentz indices respectively.) The spin connection ω_μ^{ab} is defined by $\omega_\mu^{ab} \equiv e^{\nu[a}\nabla_\mu e_\nu^{b]}$, where ∇_μ is a covariant derivative based on affine connections of spacetime $\Gamma^\rho_{\mu\nu}$ and acts on tensors. For the Dirac field, the covariant derivative based on the spin connection is defined by

$$D_\mu\psi \equiv \partial_\mu\psi - \frac{i}{4}\omega_\mu^{ab}\sigma_{ab}\psi, \quad (1)$$

where $\sigma^{ab} \equiv (i/2)[\gamma^a, \gamma^b]$ is the generator of the spinor representation of the Lorentz group and the constant γ -matrices γ^a satisfy the Clifford algebra $\{\gamma^a, \gamma^b\} = 2\eta^{ab}$; the covariant derivative acting on the Dirac adjoint $\bar{\psi} \equiv \psi^\dagger\gamma^0$ is

$$D_\mu\bar{\psi} \equiv \partial_\mu\bar{\psi} + \frac{i}{4}\omega_\mu^{ab}\bar{\psi}\sigma_{ab}. \quad (2)$$

In this paper, for explicit calculations, we will choose the Dirac representation

$$\gamma^0 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \quad \gamma^m = \begin{pmatrix} 0 & \sigma^m \\ -\sigma^m & 0 \end{pmatrix}, \quad (3)$$

where σ^m are the conventional 2×2 Pauli matrices. For later convenience, we also define the additional γ -matrix as $\gamma_5 \equiv i\gamma^0\gamma^1\gamma^2\gamma^3$. It is obvious that the covariant derivative defined by Eq. (1) transforms as a vector under diffeomorphisms and as a spinor under local Lorentz transformations. The coordinate components of γ^a are defined by

$$\gamma^\mu \equiv e^\mu_a \gamma^a, \quad (4)$$

which are shown to give a new set of γ -matrices that satisfy the algebra $\{\gamma^\mu, \gamma^\nu\} = 2g^{\mu\nu}$.

Thus, the Dirac Lagrangian generalized into a curved spacetime background is given by

$$\mathcal{L}_\psi = \frac{i}{2} [\bar{\psi}\gamma^\mu D_\mu\psi - (D_\mu\bar{\psi})\gamma^\mu\psi] - V, \quad (5)$$

where the term V generically represents a scalar potential of the Dirac field including a mass term and self-interactions, and consists of arbitrary functions of invariants generated from ψ and $\bar{\psi}$; in what follows, we assume $V = V(s)$ with $s \equiv \bar{\psi}\psi$ for simplicity.

On the other hand, for gravity, we take the Einstein-Hilbert Lagrangian

$$\mathcal{L}_g = -\frac{1}{16\pi G} e^\mu_a e^\nu_b R^{ab}_{\mu\nu}, \quad (6)$$

where $R^{ab}_{\mu\nu}$ is the curvature of the spacetime given by

$$R^{ab}_{\mu\nu} = \partial_\mu\omega_\nu^{ab} - \partial_\nu\omega_\mu^{ab} + \omega_\mu^{ac}\omega_{\nu c}^b - \omega_\nu^{ac}\omega_{\mu c}^b. \quad (7)$$

In order to take into account effects of the spin of a Dirac field on gravity, it is necessary to extend general relativity. In this paper, for this purpose, we introduce the Einstein-Cartan theory [19] in which nonvanishing torsion $C^\rho_{\mu\nu} \equiv 2\Gamma^\rho_{[\mu\nu]}$ is related with spin of matter fields. (For a very brief review of the Einstein-Cartan theory, see Appendix.)

Within this framework, we consider a system in which a Dirac field provides the unique source of metric and torsion of spacetime. Since the existence of torsion leads to the spin-interaction in the curvature (7) and the kinetic terms of the Lagrangian (5), the conventional Einstein equation is modified as

$$\tilde{G}_{\mu\nu} = 8\pi G \left(\tilde{T}_{(\mu\nu)} - \frac{3\pi G}{2} \phi_a \phi^a g_{\mu\nu} \right) \equiv 8\pi G T_{\mu\nu}^{(\text{tot})}. \quad (8)$$

The additional term $-3\pi G \phi_a \phi^a g_{\mu\nu}/2$ with $\phi^a \equiv \bar{\psi}\gamma_5\gamma^a\psi$ is the spin term prescribed by the Einstein-Cartan theory, and can be regarded as a correction due to the spin of the Dirac field. Here, the tilde indicates quantities free from torsion; $\tilde{G}_{\mu\nu}$ is the Einstein tensor composed of the Riemannian (Levi-Civita) connection; $\tilde{T}_{(\mu\nu)}$ is the usual, symmetric energy-momentum tensor of the Dirac field, given by

$$\tilde{T}_{(\mu\nu)} = \frac{i}{2} [\bar{\psi}\gamma_{(\mu}\tilde{D}_{\nu)}\psi - (\tilde{D}_{(\nu}\bar{\psi})\gamma_{\mu)}\psi] - g_{\mu\nu}\tilde{\mathcal{L}}_\psi, \quad (9)$$

where \tilde{D}_μ is the covariant derivative based on the Riemannian spin connection $\tilde{\omega}_\mu^{ab} \equiv e^{\nu[a}\tilde{\nabla}_\mu e_\nu^{b]}$, and $\tilde{\mathcal{L}}_\psi$ is the torsion-free Lagrangian of the Dirac field defined by

$$\tilde{\mathcal{L}}_\psi \equiv \frac{i}{2} \left[\bar{\psi} \gamma^\mu \tilde{D}_\mu \psi - (\tilde{D}_\mu \bar{\psi}) \gamma^\mu \psi \right] - V. \quad (10)$$

It is interesting that, in Eq. (8), the spin correction to the energy-momentum tensor of the Dirac field is proportional to both the metric $g_{\mu\nu}$ and the gravitational constant G ; such a simple form strongly motivates us to study the Dirac-gravity system including effects of the spin.

Similarly, the conventional Dirac equation in curved spacetime is also modified so as to include the spin-interaction term:

$$i\gamma^\mu \tilde{D}_\mu \psi - V' \psi + 3\pi G \phi_a \gamma_5 \gamma^a \psi = 0, \quad (11)$$

where the prime denotes the derivative with respect to s . As can be seen from Eqs. (8) and (11), one can arrive at the usual general-relativistic equations whenever the energy scale of the spin-interaction $G\phi_a\phi^a$ is negligible in comparison with typical energy scales of the kinetic term or the potential.

With the general formalism described above, we are now interested to investigate cosmology. Let us consider the flat FRW universe, in which the metric is given by

$$ds^2 = dt^2 - a^2(t) d\mathbf{x}^2, \quad (12)$$

and the vierbein is chosen to be

$$(e_\mu^a) = \text{diag}[1, a(t), a(t), a(t)], \quad (13)$$

where t is the cosmic time and $a(t)$ is the scale factor. (See Refs. [9, 10] for the nonflat FRW universe.) Then we should exclude any spatial dependence of the Dirac field for consistency with homogeneity of the spacetime: $\partial_i \psi = 0$. Consequently, we obtain the following Dirac equation:

$$\dot{\psi} + \frac{3}{2} H \psi + i\gamma^0 V' \psi - 3i\pi G \phi_a \gamma^0 \gamma_5 \gamma^a \psi = 0, \quad (14)$$

where $H \equiv \dot{a}/a$ is the Hubble parameter and the dot denotes the derivative with respect to t . From Eq. (14), the anisotropic components $\tilde{T}_{(0i)}$ can be shown to vanish, which is consistent with isotropy of the spacetime, $\tilde{G}_{0i} = 0$.

Now the energy density and pressure, which must be specified to describe cosmological dynamics, can be found from the total energy-momentum tensor defined in Eq. (8) as

$$\rho_{\text{tot}} \equiv {}^{(\text{tot})}T_0^0 = V - \frac{3\pi G}{2} \phi_a \phi^a, \quad (15)$$

$$p_{\text{tot}} \equiv -{}^{(\text{tot})}T_i^i = sV' - V - \frac{3\pi G}{2} \phi_a \phi^a. \quad (16)$$

It should be noted that $\phi_a \phi^a$ is always negative, and consequently that the spin component in Eq. (16) contributes as extra positive pressure. In terms of these variables, the cosmological evolution equations are given by

$$H^2 = \frac{8\pi G}{3} \left(V - \frac{3\pi G}{2} \phi_a \phi^a \right), \quad (17)$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3} (-2V + 3sV' - 6\pi G \phi_a \phi^a), \quad (18)$$

which are linked through the conservation law

$$\dot{\rho}_{\text{tot}} + 3H(\rho_{\text{tot}} + p_{\text{tot}}) = 0. \quad (19)$$

One can verify that the conservation law (19) is equivalent to the Dirac equation (14). Therefore, the system that consists of a Dirac field and the flat FRW spacetime is self-consistent in the Einstein-Cartan theory as well as in general relativity.

Finally, we define the equation of state w as

$$w \equiv \frac{p_{\text{tot}}}{\rho_{\text{tot}}} = \frac{sV' - V - 3\pi G \phi_a \phi^a / 2}{V - 3\pi G \phi_a \phi^a / 2}. \quad (20)$$

It is worth mentioning that, by taking the torsion-free limit $G\phi_a\phi^a/V \rightarrow 0$, we can always reproduce the cosmological equations in general relativity formulated by Armendáriz-Picón and Greene [15]. As an important example, the equation of state they found is

$$w_{\text{GR}} = s \frac{V'}{V} - 1. \quad (21)$$

III. INFLATION

A. Background

In this subsection, we show that a Dirac field can cause the de Sitter expansion of the background universe on the basis of the cosmological equations collected in the previous section.

In general relativity, from Eq. (21), one can discuss de Sitter inflation driven by a Dirac field, as done in Ref. [15]. In this case, V is assumed to be sufficiently flat because $w_{\text{GR}} \approx -1$ is guaranteed by

$$\left| \frac{d \ln V}{d \ln s} \right| \ll 1, \quad (22)$$

which is a similar requirement to the slow-roll condition for conventional scalar-field models of inflation. The condition (22) is satisfied if V is asymptotic to a positive constant for large s . For example, $\ln(1+s^n)$, $\tanh(ns)$, and $s^n/(1+s)^n$ with positive n are candidates for a potential that realizes the de Sitter expansion.

Also in our case, the flatness of V is a basic premise for considering inflation. First, in the evolution equation for $\phi_a \phi^a$,

$$\frac{d}{dt} (\phi_a \phi^a) + 6H \phi_a \phi^a + 4iV' \phi^0 \bar{\psi} \gamma_5 \psi = 0, \quad (23)$$

which is derived from Eq. (14), the flatness condition allows us to ignore the third term on the left hand side and hence leads to $\phi_a \phi^a \propto a^{-6}$. Then it follows that all the spin components, which are incorporated in the form of $G\phi_a \phi^a$ in the background equations presented previously, decrease faster than the other terms in those equations as the universe expands.

When the spin components become so small as to be negligible, all the background equations in the previous section arrive back at those of Ref. [15]. If the potential retains the sufficient flatness until that time, then the equation of state eventually becomes $w \approx w_{\text{GR}} \approx -1$. Thus, a flat potential guarantees de Sitter inflation in our model as well. In FIG. 1, we have numerically demonstrated that w approaches the value of -1 as the spin components decrease under a sufficiently flat potential.

Meanwhile, we can also speculate about the early universe before the inflation era. The result that the spin components evolve as $\phi_a \phi^a \propto a^{-6}$ also means that these become greater in earlier stages of the universe. Thus it can be seen that the very early universe is dominated by the spin components in our model, and also that, from Eq. (20), during such an early epoch the Dirac field satisfies $w \approx 1$ which corresponds to the equation of state for a stiff fluid. Therefore, our model suggests that inflation begins when the spin-dominated era ends, which can be seen also from FIG. 1. Since the Einstein-Cartan theory is expected to be relevant to high energy physics, especially theories of supergravity [34, 35] in which the algebraic equivalence between torsion and spin is necessary for the construction of supersymmetric transformations, it is acceptable that the spin components dominate the very early universe which should be considered to be at high temperature.

On the other hand, the evolution of s is given by $s \propto a^{-3}$ during inflation because Eq. (14) can be recast as

$$\dot{s} + 3Hs + 6i\pi G\phi^0 \bar{\psi}\gamma_5\psi = 0, \quad (24)$$

where the third term on the left hand side arises from the spin-interaction, and hence approximately vanishes when inflation begins. Consequently, number of e -foldings between a time t_1 and a later time t_2 is given by

$$N = \frac{1}{3} \ln \frac{s(t_1)}{s(t_2)}. \quad (25)$$

Therefore, if one requires $N \approx 60$ in order to solve the horizon problem, then s must change by approximately eighty orders of magnitude. In this study we assume the potential to be so sufficiently flat as to satisfy Eq. (22) for such a wide range of s regardless of its origin; our purpose is to investigate whether the Dirac field with a flat potential can be a source of inflation in the Einstein-Cartan theory.

Similarly to the scalar s , the pseudoscalar s_5 defined by $s_5 \equiv \bar{\psi}\gamma_5\psi$ evolves according to $s_5 \propto a^{-3}$ during inflation because s_5 fulfills the evolution equation

$$\dot{s}_5 + 3Hs_5 + 2iV'\phi^0 + 6i\pi G\phi^0 s = 0, \quad (26)$$

where both the last and the penultimate terms on the left hand side are negligible under our assumption. We also note that, in terms of the rescaled field defined by $\Psi \equiv a^{3/2}\psi$, these scalars, s and s_5 , can be rewritten as $s = a^{-3}\bar{\Psi}\Psi$, $s_5 = a^{-3}\bar{\Psi}\gamma_5\Psi$ respectively, where $\bar{\Psi}\Psi$ and $\bar{\Psi}\gamma_5\Psi$ are constant as long as the sufficient flatness of the potential holds.

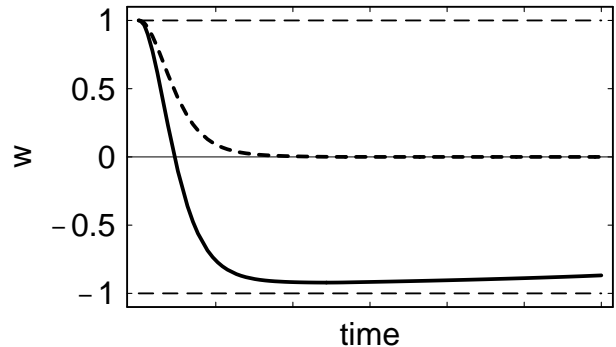


FIG. 1: Typical time evolution of the equation of state w . The solid curve represents w for the solution of the cosmological equations (14) and (18) with the potential of the form $\ln(1 + s^2)$, and shows that its value which is initially $w \approx 1$ comes close to -1 as time progresses. The dashed curve represents the simultaneous evolution of the spin components in w , $(-3\pi G\phi_a \phi^a/2)(V - 3\pi G\phi_a \phi^a/2)^{-1}$, disappearing with time.

B. Perturbation

In this subsection, we discuss perturbations of the Dirac field that brings about de Sitter inflation via the dynamics explained above.

The conventional scalar-field models of inflation [1] have an important feature of predicting a nearly scale-invariant spectrum of density fluctuations in excellent agreement with recent observations [2, 3, 4, 5]. Our next task is thus to examine the consistency of the Dirac-field model of inflation with the observations, namely, whether our model can derive a nearly scale-invariant spectrum, by computing the power spectrum of density perturbations of the Dirac field. Although a proper analysis for this purpose should be based on gauge-invariant perturbation theories [36, 37, 38, 39] where both spacetime and matter fields are perturbed, we will not consider the metric perturbations but only the perturbed field $\delta\psi$ for simplicity.

Our key strategy is to perturb the spin components of the background equations. Whereas Armendáriz-Picón and Greene [15] showed that, within the framework of general relativity, Dirac-field models of inflation with a flat potential lead to a scale-dependent spectrum and hence are inconsistent with the observations, our model based on the Einstein-Cartan theory possesses in the first place the spin-interaction, which opens the possibility of

improving their result. It should be noted that the terms related to the spin in our model appear as a consequence of a natural extension of general relativity. As discussed previously, the inflationary expansion itself can be shown to occur by simply assuming the flat potential, whether the Dirac field has the spin-interaction or not; however, if the spin terms exist, then their fluctuations must also exist. We will show below that our model predicts a nearly scale-invariant spectrum owing to these fluctuations.

In what follows, we mark the background quantities with B , and for convenience, use the identity

$$\phi_a \phi^a = -(\bar{\psi}\psi)^2 - (i\bar{\psi}\gamma_5\psi)^2 \equiv -s^2 + s_5^2. \quad (27)$$

The scalar $s = \bar{\psi}\psi$ and the pseudoscalar $s_5 = \bar{\psi}\gamma_5\psi$ are now perturbed as

$$\begin{aligned} s &= \bar{\psi}_B\psi_B + \bar{\psi}_B\delta\psi + \delta\bar{\psi}\psi_B \equiv s_B + \delta s, \\ s_5 &= \bar{\psi}_B\gamma_5\psi_B + \bar{\psi}_B\gamma_5\delta\psi + \delta\bar{\psi}\gamma_5\psi_B \equiv s_5^B + \delta s_5. \end{aligned} \quad (28)$$

The equation of motion for $\delta\psi$ is then given by

$$\begin{aligned} i\gamma^0 \left(\delta\dot{\psi} + \frac{3}{2}H\delta\psi \right) + i\frac{1}{a}\gamma^m\partial_m\delta\psi - m\delta\psi \\ - 3\pi G [(s_B - s_5^B\gamma_5)\delta\psi + (\delta s - \delta s_5\gamma_5)\psi_B] = 0, \end{aligned} \quad (30)$$

where $m \equiv V'$ and we have ignored V'' because of the flatness of V , from which it follows that m is approximately constant. Again we note that the term proportional to G is the correction due to torsion or equivalently spin; i.e., one can take the limit $G \rightarrow 0$ as a method to render the perturbed equations torsion-free in the dynamics of perturbations, while, in the background equations, the torsion-free equations are reproduced as a result of rapid disappearance of the spin components of the form $G\phi_a\phi^a$. On the other hand, taking account of the fact that general relativity provides a successful description of the present universe, one can consider the correction coming from torsion to be generally small. In other words, we can regard the term proportional to G as a first-order correction to general relativity, and will ignore higher-order corrections in G whenever the quantities derived below include such terms.

In the limit $G \rightarrow 0$, the torsion-free equation of motion is found to be

$$i\gamma^\mu \tilde{D}_\mu \delta\tilde{\psi} - m\delta\tilde{\psi} = 0, \quad (31)$$

which can be solved analytically [11]. For $H \approx \text{const.}$, the plane-wave solution $u_s(\mathbf{k}, t)$ of Eq. (31) is generally given by

$$u_s = a^{-3/2} \sqrt{\frac{-\pi k \eta}{2}} \begin{pmatrix} \alpha_k^+ H_\nu^{(1)}(-k\eta) + \beta_k^+ H_\nu^{(2)}(-k\eta) \\ \alpha_k^- H_\nu^{(1)}(-k\eta) + \beta_k^- H_\nu^{(2)}(-k\eta) \end{pmatrix}, \quad (32)$$

where η is the conformal time and $H_\nu^{(1,2)}(-k\eta)$ are the Hankel functions of the first and the second kind with $\nu = 1/2 - im/H \approx \text{const.}$; α_k^\pm and β_k^\pm are arbitrary

constant two-spinors. The solution of Eq. (31) can then be expanded as

$$\begin{aligned} \delta\tilde{\psi} &= \int \frac{d^3k}{(2\pi)^{3/2}} \sum_{s=1}^2 [u_s(\mathbf{k}, t) a_s(\mathbf{k}) e^{i\mathbf{k}\cdot\mathbf{x}} \\ &\quad + v_s(\mathbf{k}, t) b_s^\dagger(\mathbf{k}) e^{-i\mathbf{k}\cdot\mathbf{x}}], \end{aligned} \quad (33)$$

where the mode function $v_s(\mathbf{k}, t)$ corresponding to negative energy solutions is of the same form as Eq. (32).

Now, for the purpose of investigating the changes due to the spin from the torsion-free case, it is useful to introduce scalar functions $A(\mathbf{k}, t)$ and $B(\mathbf{k}, t)$, and to express the plane-wave expansion of the solution of Eq. (30) in terms of the analytical solution (32) as

$$\begin{aligned} \delta\psi &= \int \frac{d^3k}{(2\pi)^{3/2}} \sum_{s=1}^2 [A(\mathbf{k}, t) u_s(\mathbf{k}, t) a_s(\mathbf{k}) e^{i\mathbf{k}\cdot\mathbf{x}} \\ &\quad + B(\mathbf{k}, t) v_s(\mathbf{k}, t) b_s^\dagger(\mathbf{k}) e^{-i\mathbf{k}\cdot\mathbf{x}}], \end{aligned} \quad (34)$$

in which expression, A and B play a role to represent the modifications to the torsion-free case; i.e., by substituting Eq. (34) into Eq. (30), one can see that the evolution of A and B is governed by the spin-interaction part of Eq. (30), and also that both A and B are constant in the limit $G \rightarrow 0$. Here, a_s and b_s are the particle and antiparticle operators satisfying $\{a_s(\mathbf{k}), a_{s'}^\dagger(\mathbf{k}')\} = \{b_s(\mathbf{k}), b_{s'}^\dagger(\mathbf{k}')\} = \delta_{ss'} \delta^3(\mathbf{k} - \mathbf{k}')$ under an appropriate orthonormality. Then, the vacuum expectation value of the square of $\delta\psi$ is

$$\langle \delta\bar{\psi}\delta\psi \rangle = \int \frac{d^3k}{(2\pi)^3} |B|^2 \sum_s \bar{v}_s(\mathbf{k}) v_s(\mathbf{k}). \quad (35)$$

Next, we characterize density perturbations $\delta\rho$ by the variable

$$\zeta \equiv \frac{\delta\rho}{\rho + p}, \quad (36)$$

This quantity is the one employed in Ref. [15], defined analogously to the gauge-invariant Bardeen variable, which is conserved for adiabatic perturbations on sufficiently large scales [36, 40]. For our model, we have $\rho + p = m s_B$ and

$$\delta\rho = m\delta s + 3\pi G(s_B\delta s - s_5^B\delta s_5), \quad (37)$$

from which we compute the power spectrum \mathcal{P} of the variable ζ on comoving scales $1/k$, defined by

$$\langle \zeta(\mathbf{x}, t) \zeta(\mathbf{x} + \mathbf{r}, t) \rangle = \int \frac{dk}{k} \frac{\sin kr}{kr} \mathcal{P}(k). \quad (38)$$

We have supposed that the Fourier transform of the correlation function on the left hand side depends only on k and not on \mathbf{k} itself, for consistency with the isotropy of the background spacetime.

In order to evaluate the isotropic power spectrum of $\langle \zeta^2 \rangle$, following the method of Ref. [15], we apply the Fierz

transformation to $\langle \zeta^2 \rangle$ and expand the vacuum expectation values of the quadratic term of δs and δs_5 in terms of perturbation bilinears. For example, the vacuum expectation value of δs^2 can be expressed as

$$\langle \delta s^2 \rangle = \frac{s_B}{2} \langle \delta \bar{\psi} \delta \psi \rangle + \frac{\bar{\psi}_B \gamma^a \psi_B}{2} \langle \delta \bar{\psi} \gamma_a \delta \psi \rangle + \dots \quad (39)$$

After that, as noted in Ref. [15], we concentrate on the perturbations of the scalar bilinear form $\langle \delta \bar{\psi} \delta \psi \rangle$ and discard the remaining vectorial bilinears that may break isotropy of the power spectrum because we are only concerned with determining the amplitude and k -dependence of \mathcal{P} . In this way, \mathcal{P} is obtained as

$$\mathcal{P}(k) = \frac{|B|^2 k^3}{4\pi^2 \bar{\Psi}_B \Psi_B} \left(1 + \frac{3\pi G C_1 a^{-3}}{m} \right) \sum_s \bar{V}_s V_s, \quad (40)$$

where V_s is the rescaled mode function defined by $V_s \equiv a^{3/2} v_s$ and $C_1 = 2\bar{\Psi}_B \Psi_B - (\bar{\Psi}_B \gamma_5 \Psi_B)^2 / \bar{\Psi}_B \Psi_B$ is a constant. We have here ignored the terms of $O(G^2)$ as mentioned before. In comparison with the result of Ref. [15], the extra factor $|B|^2$ appears in addition to the first-order correction in the parentheses in the above equation. The effective part of B that contributes to the isotropic power spectrum fulfills the following equation:

$$\frac{\dot{B}}{B} = - \frac{3i\pi G}{\sum_s \bar{V}_s V_s} \sum_s (s_B \bar{V}_s \gamma^0 V_s - s_5^B \bar{V}_s \gamma^0 \gamma_5 V_s + \bar{\psi}_B V_s \bar{V}_s \gamma^0 \psi_B - \bar{\psi}_B \gamma_5 V_s \bar{V}_s \gamma^0 \gamma_5 \psi_B), \quad (41)$$

which is obtained by multiplying Eq. (30) by $\delta \bar{\psi}$ with Eq. (34) and evaluating in a vacuum.

Now we have to determine the mode function V_s and B . We here take the Bunch-Davies vacuum as an appropriate initial vacuum state [33], and choose the constants in V_s so that V_s coincides with the Minkowski solution $V_s \propto e^{ik\eta}$ on sufficiently short scales $k \rightarrow \infty$:

$$V_s = \frac{\sqrt{-\pi k \eta}}{2} \begin{pmatrix} e^{-\pi m/2H} H_\nu^{(2)}(-k\eta) \frac{\mathbf{k} \cdot \boldsymbol{\sigma}}{k} \omega^s \\ e^{\pi m/2H} H_{\nu^*}^{(2)}(-k\eta) \omega^s \end{pmatrix}, \quad (42)$$

where ω^s is the constant two-spinor defined by $\omega_\alpha^s = \delta_\alpha^s$. With this choice, in order to solve Eq. (41), we use the explicit background solutions during inflation,

$$a = a_* e^{Ht} \quad (43)$$

$$\Psi_B = \begin{pmatrix} \varphi_0 e^{-imt} \\ \chi_0 e^{imt} \end{pmatrix}, \quad (44)$$

where φ_0 and χ_0 are arbitrary constants. Moreover, fortunately, we can make use of the fact that $\sum_s \bar{V}_s V_s$ is approximately constant with respect to $z = -k\eta$. Roughly speaking, the relation $|d(\sum_s \bar{V}_s V_s)/dz| \ll |\sum_s \bar{V}_s V_s|$ holds for $z > 1$ and near $z = 0$ independently of m . In fact, the asymptotic formulae of the Hankel functions [41] lead to $\sum_s \bar{V}_s V_s \sim -2(m/H)z^{-1}$ for large z and $\sum_s \bar{V}_s V_s \sim -2 \tanh(\pi m/H)$ for small z . We can safely employ this relation because, as discussed below, we only need to find the asymptotic behavior of $|B|^2$ on large and small scales. Then Eq. (41) can be integrated to give

$$\begin{aligned} |B|^2 \approx & 1 + \frac{3i\pi^2 G H^2}{2 \sum_s \bar{V}_s V_s k^3} \left\{ \left(\frac{k}{a_* H} \right)^{2im/H} \frac{\varphi_0^\dagger \chi_0 z^3}{\cosh^2 \frac{\pi m}{H}} \right. \\ & \times \left[-i \sinh \frac{\pi m}{H} \Gamma \left(\nu + \frac{3}{2} \right) z^{2\nu} {}_2F_3 \left(\frac{3}{2}, \nu + \frac{3}{2}; \nu + 1, \nu^* + 1, \nu + \frac{5}{2}; -z^2 \right) \right. \\ & - \frac{z^{-4im/H}}{\sqrt{\pi}} \Gamma \left(\nu + \frac{1}{2} \right) \Gamma \left(2\nu + \frac{1}{2} \right) {}_2F_3 \left(\nu + \frac{1}{2}, 2\nu + \frac{1}{2}; \nu, 2\nu, 2\nu + \frac{3}{2}; -z^2 \right) \\ & \left. \left. + \frac{1}{2} \Gamma \left(\nu^* + \frac{1}{2} \right) {}_2F_3 \left(\frac{3}{2}, \nu^* + \frac{1}{2}; \frac{5}{2}, \nu^*, 2\nu^*; -z^2 \right) \right] - h.c. + C_2 \right\}, \end{aligned} \quad (45)$$

where ${}_2F_3(a_1, a_2; b_1, b_2, b_3; x)$ is the regularized hypergeometric function defined by

$$\begin{aligned} & {}_2F_3(a_1, a_2; b_1, b_2, b_3; x) \\ & \equiv \frac{1}{\Gamma(b_1)\Gamma(b_2)\Gamma(b_3)} \sum_{k=0}^{\infty} \frac{(a_1)_k (a_2)_k}{(b_1)_k (b_2)_k (b_3)_k} \frac{x^k}{k!}, \end{aligned} \quad (46)$$

with the Pochhammer symbol $(a)_k \equiv \Gamma(a+k)/\Gamma(a)$, and we have dropped the higher-order terms in G . The integration constant C_2 should be chosen so that $|B|^2 \rightarrow 1$ on sufficiently short scales $k \rightarrow \infty$. Using the asymptotic

expansion of ${}_2F_3(a_1, a_2; b_1, b_2, b_3; x)$ for $|x| \rightarrow \infty$,

$$\begin{aligned} & {}_2F_3(a_1, a_2; b_1, b_2, b_3; x) \\ & \sim \frac{(-x)^c}{2\sqrt{\pi}\Gamma(a_1)\Gamma(a_2)} \left(e^{-i(\pi c + 2\sqrt{-x})} + e^{i(\pi c + 2\sqrt{-x})} \right) \\ & \quad + \frac{\Gamma(a_2 - a_1)(-x)^{-a_1}}{\Gamma(b_1 - a_1)\Gamma(b_2 - a_1)\Gamma(b_3 - a_1)\Gamma(a_2)} \\ & \quad + \frac{\Gamma(a_1 - a_2)(-x)^{-a_2}}{\Gamma(b_1 - a_2)\Gamma(b_2 - a_2)\Gamma(b_3 - a_2)\Gamma(a_1)}, \end{aligned} \quad (47)$$

with $c = (a_1 + a_2 - b_1 - b_2 - b_3 + 1/2)/2$, we have

$$C_2 = \text{sech} \frac{2\pi m}{H} \left[\frac{\varphi_0^\dagger \chi_0 \Gamma(-\nu)}{\Gamma(-\nu - \frac{1}{2}) \Gamma(\frac{1}{2} - 2\nu)} \left(\frac{k}{a_* H} \right)^{2im/H} - \frac{\chi_0^\dagger \varphi_0 \Gamma(-\nu^*)}{\Gamma(-\nu^* - \frac{1}{2}) \Gamma(\frac{1}{2} - 2\nu^*)} \left(\frac{k}{a_* H} \right)^{-2im/H} \right]. \quad (48)$$

The power spectrum \mathcal{P} outside horizon is obtained

$$\begin{aligned} \mathcal{P}|_{k=aH} \approx & -\frac{GH^2}{\Psi_B \Psi_B} \left(\frac{3}{2} C_1 + \frac{m}{H} (\varphi_0^\dagger \chi_0 + \chi_0^\dagger \varphi_0) \left(\frac{1}{3\pi} (3\gamma + \ln 8 - 1) - \frac{3}{8} \right) \right. \\ & - \frac{m^2}{H^2} \left\{ \frac{\pi^2}{2} C_1 + i(\varphi_0^\dagger \chi_0 - \chi_0^\dagger \varphi_0) \left[\left(\frac{3}{4} - \frac{2}{3\pi} (3\gamma + \ln 8 - 1) \right) \ln \left(\frac{k}{a_* H} \right) \right. \right. \\ & \left. \left. - \frac{2}{9\pi} (3\gamma + \ln 8 - 2) + \frac{3}{8} (2\gamma + \ln 4 - 3) \right] \right\} \right), \end{aligned} \quad (49)$$

where γ is Euler's constant, $\gamma \approx 0.577$. Therefore, the spectral index is

$$n - 1 = \frac{m^2}{H^2 C_1} i(\varphi_0^\dagger \chi_0 - \chi_0^\dagger \varphi_0) \left(\frac{4}{9\pi} (3\gamma + \ln 8 - 1) - \frac{1}{2} \right), \quad (50)$$

which is our final result. It should be emphasized that the k -dependence of \mathcal{P} is naturally suppressed by the condition $m/H \ll 1$ which follows from the flatness of the potential, and hence that n is nearly equal to 1. We also note that the running index $\alpha = dn/d \ln k$ is further suppressed, of the order $O(m^3)$. Therefore, we are led to the conclusion that our Dirac-field model of inflation can predict a nearly scale-invariant spectrum of density fluctuations in agreement with the observations.

IV. SUMMARY

We have shown that the Dirac-field model of inflation leads to a nearly scale-invariant spectral index consistent with the observations, by naturally extending the theoretical framework beyond general relativity. It is usually believed that general relativity does not hold in the very early universe and then should be extended appropriately to high energy physics; the Einstein-Cartan theory adopted here is one of such extended theories.

In the framework of general relativity, Armendáriz-Picón and Greene [15] found that a Dirac field with a flat potential can give rise to the de Sitter expansion of the universe, but concluded that the Dirac field itself cannot be an alternative to the conventional inflaton field as the unique source of inflation because the spectral index obtained from density fluctuations of the Dirac field, $n = 4$, is in strong disagreement with the observations. The key

by evaluating Eq. (40) on large scales $k \rightarrow 0$ with Eq. (45), in which we can use the asymptotic formula ${}_2F_3(a_1, a_2; b_1, b_2, b_3; x) \sim (\Gamma(b_1)\Gamma(b_2)\Gamma(b_3))^{-1}$ for $x \rightarrow 0$. Since the observed spectral index at horizon crossing is defined by $n - 1 \equiv d \ln \mathcal{P} / d \ln k|_{k=aH}$, we subsequently estimate \mathcal{P} at $k = aH$. For simplicity, let us here utilize an approximation $m/H \ll 1$, which is allowed by the flatness of V . Thus we find

ingredient for improving their result, namely, obtaining $n \approx 1$, is to introduce a spin-interaction, which naturally appears in the Einstein-Cartan theory, into the dynamics of the Dirac field that has the inflationary potential.

Because of the existence of the spin-interaction, the new terms of the form $G\phi_a\phi^a$ must be added to both the cosmological background equations and the equation of motion for the perturbed field presented in Ref. [15]. However, we have seen that, according to Eq. (23), the spin terms in the background equations decrease fast as the universe expands; therefore, during inflation, we can ignore the effects due to the spin on the background. Without any change in the basic idea that a flat potential of a Dirac field leads to an inflationary expansion, we have been able to gain the possibility to solve the problem of the spectral index, which is an issue in perturbation theories, not in background dynamics.

The spectral index obtained in our model, $n = 1 + O(m^2)$, is nearly scale-invariant by virtue of the flatness of the potential, which is similar to the situation in the conventional inflaton models. In this regard, so far, the Dirac-field model does not provide such novel features as the typical inflaton models do not have. However, it is important to recognize that a Dirac field can drive inflation of the universe. As is well known, the spinor fields are indispensable not only in the description of relativistic quantum fields, but also in the context of supersymmetric unification of all fundamental interactions at high energy scales. Therefore, in the construction of realistic cosmological models containing various matter fields and interactions between them, attention should be paid to the behavior or properties of the spinor fields that significantly affect the geometry of spacetime and consequently can have a central role in the evolution of the universe.

APPENDIX: EINSTEIN-CARTAN THEORY

For completeness, in this appendix we briefly review the Einstein-Cartan theory [19]. The Einstein-Cartan theory is a natural extension of Einstein's gravity theory, and is one of theories that give a dynamical role to both spin and mass of matter.

The spacetime in this theory is described by Riemann-Cartan geometry known as a generalization of Riemann geometry to include torsion. In the Riemann-Cartan geometry, from asymmetric affine connections $\Gamma^\rho_{\mu\nu}$, the covariant derivative for tensors is defined by $\nabla_\nu V^\mu = \partial_\nu V^\mu + \Gamma^\mu_{\rho\nu} V^\rho$. The connection is constrained by the metricity condition $\nabla_\rho g_{\mu\nu} = 0$, which is postulated in order for a local Minkowski structure to be guaranteed. The curvature tensor is constructed from such connections as

$$R^\rho_{\sigma\mu\nu} \equiv \partial_\mu \Gamma^\rho_{\sigma\nu} - \partial_\nu \Gamma^\rho_{\sigma\mu} + \Gamma^\rho_{\lambda\mu} \Gamma^\lambda_{\sigma\nu} - \Gamma^\rho_{\lambda\nu} \Gamma^\lambda_{\sigma\mu}. \quad (\text{A.1})$$

The difference from the Riemann geometry is that $\Gamma^\rho_{\mu\nu}$ is asymmetric. Actually, if we demand the connection be symmetric, it can be fixed as the well-known Riemannian connection

$$\tilde{\Gamma}^\rho_{\mu\nu} = \frac{1}{2} g^{\rho\sigma} (\partial_\nu g_{\sigma\mu} + \partial_\mu g_{\sigma\nu} - \partial_\sigma g_{\mu\nu}). \quad (\text{A.2})$$

There is, however, no *a priori* reason that we suppose $\Gamma^\rho_{\mu\nu}$ to be symmetric in general. In the Riemann-Cartan geometry, the antisymmetric part is kept as

$$C^\rho_{\mu\nu} \equiv 2\Gamma^\rho_{[\mu\nu]} \equiv \Gamma^\rho_{\mu\nu} - \Gamma^\rho_{\nu\mu}, \quad (\text{A.3})$$

which can be shown to transform as a tensor, a purely geometrical quantity. Since the infinitesimal parallelograms do not close in this spacetime and the closure failure is proportional to $C^\rho_{\mu\nu}$, this tensor serves as the torsion of the spacetime.

These geometrical quantities, the curvature and the torsion, can be understood from the local Poincaré gauge theory [19, 21, 42, 43, 44] in which vierbeins e^μ_a and spin connections ω_μ^{ab} are introduced as the gauge fields of the theory. In terms of the gauge fields, we can define the translational field strength corresponding to the torsion as well as the rotational field strength corresponding to the curvature [45].

From the correspondence between a coordinate basis and a tetrad, the vierbein satisfies

$$\partial_\nu e^{\mu a} + \Gamma^\mu_{\rho\nu} e^{\rho a} + \omega_\nu^{ab} e^\mu_b = 0. \quad (\text{A.4})$$

The above equation guarantees a conversion $\nabla_\mu V^\nu = e^\nu_a D_\mu V^a$, where D_μ is the Lorentz covariant derivative based on ω_μ^{ab} .

By virtue of the metricity condition, $\Gamma^\rho_{\mu\nu}$ can be decomposed into the Riemannian piece and the non-Riemannian piece as

$$\Gamma^\rho_{\mu\nu} = \tilde{\Gamma}^\rho_{\mu\nu} + \frac{1}{2} (C^\rho_{\mu\nu} + C_{\mu\nu}{}^\rho + C_{\nu\mu}{}^\rho), \quad (\text{A.5})$$

from which it is clear that the connection reduces to the Riemannian one when torsion vanishes. Such decomposition holds also for the spin connection with the aid of Eq. (A.4).

The basic field equations of the Einstein-Cartan theory are derived from the simplest Lagrangian of a gravity-matter system

$$\mathcal{L}(e^\mu_a, \omega_\mu^{ab}, \varphi, \partial_\mu \varphi) = -\frac{R}{16\pi G} + \mathcal{L}_m(e^\mu_a, \varphi, D_\mu \varphi), \quad (\text{A.6})$$

where $R \equiv e^\mu_a e^\nu_b R^{ab}_{\mu\nu}$ is the scalar curvature in the Riemann-Cartan spacetime and φ generically represents matter fields minimally coupled to gravity. Here, by making use of the relation (A.4), not only the matter Lagrangian \mathcal{L}_m , but also the Einstein-Hilbert Lagrangian can be expressed as a function of e^μ_a and ω_μ^{ab} .

For a derivation of the field equations, it is useful to adopt the so-called Palatini approach where e^μ_a and ω_μ^{ab} are treated as independent variables. Varying the Lagrangian (A.6) with respect to both, one obtains

$$G_\mu{}^a \equiv R_\mu{}^a - \frac{1}{2} e_\mu{}^a R = 8\pi G T_\mu{}^a, \quad (\text{A.7})$$

$$\begin{aligned} C^\mu_{ab} + 2e^\mu_{[a} C^\nu_{b]\nu} &= -8\pi G i \frac{\partial \mathcal{L}_m}{\partial D_\mu \varphi^A} (S_{ab})^A{}_B \varphi^B \\ &\equiv -16\pi G S^\mu_{ab}, \end{aligned} \quad (\text{A.8})$$

where $R_\mu{}^a \equiv e^\nu_b R^{ba}_{\nu\mu}$ is the asymmetric Ricci tensor and S_{ab} is the generator of the Lorentz group. (The capital indices are spacetime or spinor indices.)

One of the resulting equations, Eq. (A.7), is a generalized version of the familiar Einstein equation in the sense that the Einstein tensor $G_\mu{}^a$ which consists of the metric-compatible and asymmetric connection is related with the canonical energy-momentum tensor $T_\mu{}^a$. The other equation (A.8) exhibits that the spin density S^μ_{ab} of matter induce torsion of the spacetime.

Since the second field equation (A.8) is algebraic, we can substitute everywhere spin for torsion. Then, according to Eq. (A.5), the non-Riemannian part of the covariant derivatives in \mathcal{L}_m produces an interaction between S^μ_{ab} and φ , called a spin-interaction, in the equation of motion for φ .

Moreover, Eq. (A.5) can also be applied to splitting the Einstein tensor $G_\mu{}^a$ defined by Eq. (A.7) into the Riemannian piece and the non-Riemannian piece. Therefore, all the terms including the torsion in Eq. (A.7) can be interpreted as a spin correction to the usual, symmetric energy-momentum tensor $\tilde{T}_{(\mu\nu)}$ through the following expression:

$$\tilde{G}_{\mu\nu} = 8\pi G \left(\tilde{T}_{(\mu\nu)} + T_{\mu\nu}^{(\text{spin})} \right), \quad (\text{A.9})$$

where $\tilde{G}_{\mu\nu}$ is the Einstein tensor composed of the Riemannian connection $\tilde{\Gamma}^\rho_{\mu\nu}$. The explicit form of $T_{\mu\nu}^{(\text{spin})}$ is

given by

$$T_{\mu\nu}^{(\text{spin})} = 8\pi G \left[S_{\mu}^{\rho\sigma} S_{\nu\rho\sigma} + 2S_{\mu\rho}^{\rho} S_{\nu\sigma}^{\sigma} - 4S^{(\rho\sigma)}_{\mu} S_{(\rho\sigma)\nu} + \frac{1}{2}g_{\mu\nu} \left(4S^{(\rho\sigma)\lambda} S_{(\rho\sigma)\lambda} - S^{\rho\sigma\lambda} S_{\rho\sigma\lambda} - 2S^{\sigma\rho}_{\sigma} S^{\lambda}_{\rho\lambda} \right) \right] + 2 \left(\nabla_{\rho} - 8\pi G S^{\sigma}_{\rho\sigma} \right) S_{(\mu\nu)}^{\rho} + T_{(\mu\nu)}^{(\text{kin})}, \quad (\text{A.10})$$

where $T_{\mu\nu}^{(\text{kin})}$ arises from the coupling of φ to S^{μ}_{ab} in the covariantized kinetic terms of \mathcal{L}_m .

The Einstein-Cartan theory is constructed in this way. Since, as can be seen from the above equation, the spin squares contribute to the energy-momentum in the form proportional to G , the predictions of the Einstein-Cartan theory deviate from those of general relativity only when the matter fields are coupled to gravity at high energy scales. Therefore, it is reasonable to take the Einstein-Cartan theory as a suitable framework for considering early stages of the universe, in which general relativity is usually not believed to be valid.

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